# KDAC Equation (8)

## 1 Introduction

This is the deduction of equation 8 for the KDAC paper at `http://people.eecs.berkeley.edu/~jordan/papers/niu-dy-jordan-pami.pdf`

$$\Phi(\alpha) = f(w_{k+1}),$$

where

$$w_{k+1} = \sqrt{1-\alpha^2}w_k + \alpha\nabla f_{\perp}$$

and

$$\begin{aligned} f(w_k) &= \sum_{ij} \gamma_{ij} k(w_k x_i, w_k x_j) \\ &= \sum_{ij} \gamma_{ij} exp(-\frac{w_1^T A_{ij} w_1}{2\sigma^2}) exp(-\frac{w_2^T A_{ij} w_2}{2\sigma^2}) \dots exp(-\frac{w_k^T A_{ij} w_k}{2\sigma^2}) \end{aligned}$$

Derivative of $\Phi(\alpha)$ is:

$$\begin{aligned} \Phi'(\alpha) = \frac{\partial \Phi(\alpha)}{\partial \alpha} &= \nabla f(w_{k+1})^T \frac{\partial w_{k+1}}{\partial \alpha} \\ &= \nabla f(w_{k+1})^T [\frac{1}{2}(1-\alpha^2)^{-\frac{1}{2}}(-2\alpha)w_k + \nabla f_{\perp}] \\ &= \nabla f(w_{k+1})^T [\frac{-\alpha}{\sqrt{1-\alpha^2}} w_k + \nabla f_{\perp}], \end{aligned}$$

when $\alpha = 0$

$$\Phi'(0) = \nabla f(w_k)^T \nabla f_{\perp},$$

where

$$\nabla f(w_k) = \sum_{ij} -\gamma_{ij} \frac{1}{\sigma^2} g(w_1) g(w_2) \dots g(w_{k-1}) exp(-\frac{w_k^T A_{ij} w_k}{2\sigma^2}) A_{ij} w_k,$$

where

$$g(w_k) = exp(-\frac{w_k^T A_{ij} w_k}{2\sigma^2})$$